MONEY DEMAND AND SEIGNIORAGE-MAXIMISING INFLATION IN IRAN

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1 Introduction

This paper examines the relationship between seigniorage and inflation in Iran by using a per capita money demand function constructed based on the cash-in-advance framework.

Seigniorage is the raising of revenue by money creation, and can be used by the government to finance expenditure when taxes cannot be raised from other sources. In Iran, due to the poorly developed domestic financial markets and the restrictions on foreign borrowing, the government could not finance the budget deficit through sources other than monetisation: the budget deficit is financed through borrowing from the central bank, which increases money supply. This kind of financing is called seigniorage.

Following Lucas and Stocky (1983), and Braun (1994), in this paper, a money demand function is developed and designed using the cash-in-advance model.[1] Cash-in-advance model is primarily proposed by Clower (1967). Lucas and Stocky (1983) considered the framework in an empirical setting. The model is extended by Singleton (1985), Eckstein and Leiderman (1988), Chari, Christiano and Kehoe (1991), Hodrick, Kocherlakota, and Lucas (1991), Braun (1994), and Sill (1998).[2]

Moreover, some other issues are examined using the proposed model: whether a Laffer curve relationship exists between seigniorage revenue and the rate of inflation; the rate of inflation that maximises seigniorage revenue; model stability under adaptive and rational expectations; and the speed of agents' response to shocks, such

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[1] Three frameworks have been used to design a money demand function. They comprise 'shopping time' transaction technology, money in the utility function, and cash–in–advance models.
[2] For details of the literature review on the cash-in-advance model see, for example, Moradi (1999).

as an unanticipated increase in the inflation rate, in adjusting their holding of real balances.

The evidence shows that the actual rate of inflation generally exceeded the corresponding rates that would maximise seigniorage revenue in Iran after 1973. This means that the government could have obtained extra seigniorage with a lower rate of inflation. The estimated model can be used to determine the rate of inflation that would maximise seigniorage in future using output and population growth.

The structure of the paper is as follows: Section two analyses a basic model of seigniorage and inflation. Section three presents the microfoundation of the demand function for money constructed based on the cash-in-advance framework and also derives the seigniorage-maximising inflation rate. Section four focuses on the empirical analyses of seigniorage and inflation, and section five concludes.

2 Seigniorage and Inflation: A Basic Analysis

In developed economies governments tend to resort little to seigniorage and rely, instead, on taxation and bond sales, or borrowing, to finance their expenditures [see Fischer, (1982)]. But it may be more important for countries which do not rely on bonds, such as Iran, or which have less well–developed tax systems. Since money creation is associated with inflation, it is important to examine the relationship between inflation and seigniorage. The famous analysis of Cagan (1956) is first considered and then the analysis is used the cash-in-advance model to better fit the Iranian case.

2.1 Some Terminology

Before examining Cagan's model it will be useful to define some terminology such as seigniorage, inflation tax, Laffer curve, and adaptive and rational expectations.

Seigniorage[3] is defined as the real revenues of a government acquired by printing new money. In this study, the following conventional definition of seigniorage is used:[4]

[3] The term of seigniorage comes from *seigneur*. This French word was used for feudal lord in the Middle ages. The feudal lord had monopoly right on his land to coin money, while this right belongs to the government today [see, for example, Mankiw (1997)].
[4] See, for example, Friedman (1971), Bruno and Fischer (1990), Blanchard and Fischer (1994), and Obstfeld and Rogoff (1996).

$$S = \frac{\Delta M}{P} = \mu \frac{M}{P} = \mu m \qquad (1)$$

where S is seigniorage; M is aggregate nominal money balances; P is the price level; m is aggregate real money balances; and $\mu = \frac{\Delta M}{M}$ is the growth rate of aggregate nominal money.

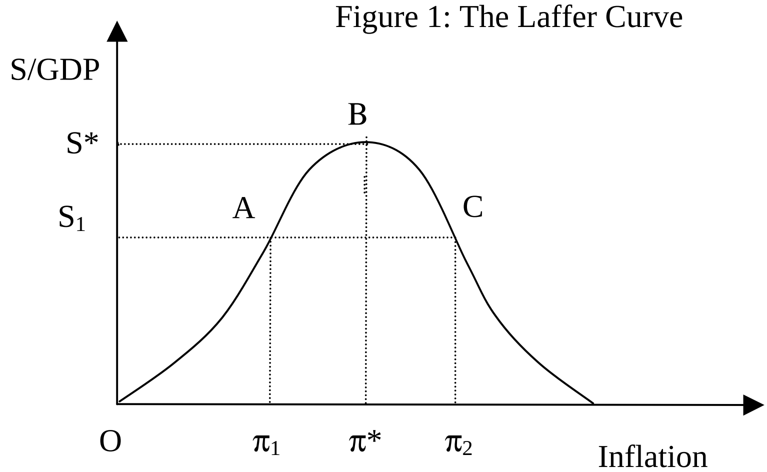
The inflation tax refers to the total loss in the value of real balances in the face of inflation, in other words, this is equal to the real depreciation of the public's cash holdings. The inflation tax can be expressed formally as

$$IT = \pi \frac{M}{P} = \pi m \qquad (2)$$

where IT represents inflation tax; and π is the rate of inflation. It is common to interpret π as the inflation tax rate and m as the tax base. In equilibrium, the inflation tax and seigniorage are equal, since $\pi = \mu$.

The concept of the Laffer curve was originally introduced to analyse the relationship between taxes paid and the rate of tax. This concept can be applied to monetary policy. In this case, the Laffer curve shows the relationship between seigniorage revenue and the rate of inflation. Figure 1 represents the Laffer curve in the context of monetary policy. The horizontal axis measures the rate of inflation and the vertical axis measures seigniorage revenue as a percentage of GDP.

Figure 1: The Laffer Curve



Initially as the rate of creation of money and the rate of inflation rises, seigniorage also increases. At point B seigniorage revenue is at a maximum, S*, corresponding to an inflation rate of π*. At any higher rate of money creation, the total seigniorage revenue will decline while the rate of inflation increases. This happens because, with the higher rate of inflation people like to hold less money than before, since they choose to avoid the inflation tax and hence their real balances are lower. At C, for example, with $\pi_2 > \pi_1$, revenue is the same as at A; although the tax rate is higher with a higher inflation rate, the tax base, m, is lower at C than at A.

Two forms of inflation expectations are considered, adaptive expectations (or error learning) and rational expectations. Under adaptive expectations, economic agents revise their expectations based on the most recent error. In particular, agents'expectations change according to:

$$\frac{d\pi^e}{dt} = \beta(\pi - \pi^e) \qquad 0 < \beta \leq 1 \tag{3}$$

where β reflects the speed of revision of expectations. This expression says that agents revise their expectations each period by a fraction, β, of the forecast error. So if π exceeds π^e, the expected rate of inflation increases. Under rational expectations, economic agents generally do not waste information and so, expectations are based on the structure of the entire system. This can be written formally as:

$$\pi^e = E(\pi \mid \Phi_{t-1}) \tag{4}$$

where E is the mathematical expectations operator; and Φ_{t-1} is all the information available at time t-1.

2.2 Cagan's Model and Seigniorage–Maximisation

Cagan examined the hyperinflation experience in several European economies in the first half of the last century. Under short periods of hyperinflation,[5] he was able to assume that changes in real variables, such as population, output and the real interest rate, were negligible relative to the monetary changes. This simplification helps to focus on monetary matters but will be relaxed later.

Given this simplification, Cagan used an aggregate demand for money function of the following form:

$$m^d = (\frac{M}{P})^d = c\exp(-\alpha\pi^e) \qquad c, \alpha > 0 \tag{5}$$

The higher the expected inflation, the lower the real demand for money balances since agents want to avoid the inflation tax. Cagan assumed that in a hyperinflation environment the change in the price level ensures that desired and actual cash balances are continuously equated. He also assumed that π^e is formed according to the adaptive expectations hypothesis.

The money demand function affects the calculation of the seigniorage–maximising inflation rate. Using the equilibrium condition, $M^d = M^s = M$, equation (5) yields:

$$M = Pc\exp(-\alpha\pi^e) \tag{6}$$

Differentiating this with respect to time after taking logarithms yields:

$$\frac{\dot{M}}{M} - \frac{\dot{P}}{P} = \mu - \pi = -\alpha\dot{\pi}^e \tag{7}$$

In steady state, with $\dot{m} = \dot{\pi} = 0$, (7) yields:

$$\pi^e = \pi = \mu \tag{8}$$

Substituting the real money demand function into the seigniorage equation (1) yields:

$$S = \mu c\exp(-\alpha\pi^e) \tag{9}$$

Using the steady state condition (8), the maximum steady state seigniorage revenue, S^*, can be obtained by:

$$S^* = \max_{\{\pi\}} \pi c\exp(-\alpha\pi) \tag{10}$$

Accordingly, the rate of monetary growth that maximises seigniorage is:

$$\mu^* = \pi^* = \frac{1}{\alpha} \tag{11}$$

This shows that the revenue–maximising net rate of money growth is simply the inverse of the semi–elasticity with respect to inflation, α, in the demand for real money function.

2.3 Dual Equilibrium and Inflation Expectations

Since the rate of inflation that generates enough seigniorage revenue to finance the deficit depends on money demand, and this varies with inflationary expectations, the price level path depends on how expectations are formed. The stability properties of the system also depend upon inflation expectations and upon whether one assumes expectations to be formed adaptively or rationally. This section examines such issues to compare the behaviour of the economy under adaptive and rational expectations.

[5] He arbitrarily defined hyperinflation to be inflation of more than 50 percent per month.

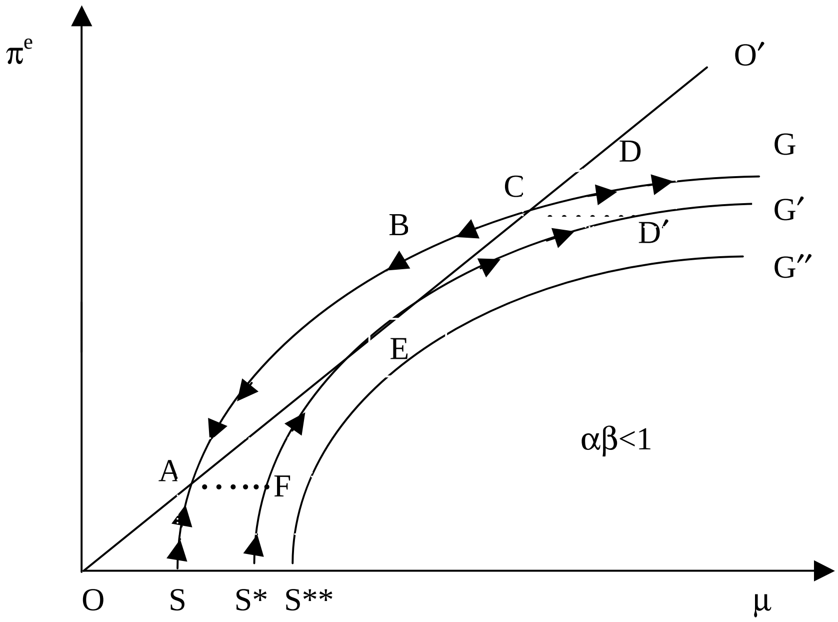
Adaptive Expectations

Consider that a government wishes to finance a fixed amount of real expenditure using seigniorage. For a given level of seigniorage, equation (9) may be written as follows:

$$\pi^e = \frac{1}{\alpha}\ln(\mu c) - \frac{1}{\alpha}\ln S \qquad (12)$$

For a given value of *S*, equation (12) shows the relationship between expected inflation, π^e, and the rate of monetary growth, μ. This relationship yields the iso–seigniorage or G–curves plotted in Figure 2, each curve showing a positive relationship between the expected rate of inflation and the growth rate of money. The vertical axis of Figure 2 measures the expected rate of inflation and the horizontal axis measures the growth rate of money. A change in seigniorage shifts the G–curve to the right or left. A rise in seigniorage revenue leads to a movement to the right, while a fall leads to a leftward movement.

Figure 2: Dynamics of Inflation with Fixed Seigniorage: under adaptive expectations



The figure shows three G–curves, G, G′ and G″ and the steady–state line, OO′. In steady state, the growth rate of money, μ, is equal to the expected rate of inflation, π^e, which is shown by the 45–degree line. Consider the curve G. The intersections of this curve and the 45–degree line give two steady state values of the growth rate of money. Similarly, note that the curve G′ yields a unique steady state, whilst the curve G″ yields no steady state. Thus, it is possible to have two, one or no steady state inflation rates depending on the level of seigniorage. The maximum level of seigniorage consistent with steady state inflation, S^*, underlies the curve G′. For a lower level of seigniorage, such as that underlying the curve G, there are two steady states (as shown at points A and C on the figure), and for a higher level of seigniorage there is no steady state.

To understand the stability of the system, substitute π from equation (7) into (3) to yield:

$$\dot{\pi}^e = \frac{\beta}{(1-\alpha\beta)}(\mu - \pi^e) \qquad (13)$$

First, consider the case when $\alpha\beta < 1$. In this case examination of (13) shows that if the economy starts from any point above the 45–degree line, for example point B on the curve G, where $\mu \prec \pi^e$, the expected rate of inflation is falling. If the economy starts from any point below the 45–degree line, for example point D on the curve G, where $\mu \succ \pi^e$, expected inflation is rising. The arrows shown on the curve G, therefore, show the direction of inflation and indicate that the steady state at A is locally stable and that at C is unstable.

At both steady state points on curve G, the government earns the same level of seigniorage revenue. But point A corresponds to large real balances with a lower rate of inflation, while point C corresponds to small real money balances with a higher rate of inflation. We assume that point A is preferable since it yields the same revenue at a lower inflation rate.[6]

If the coefficient of adaptive expectations or the elasticity of the demand function with respect to inflation is sufficiently large, so that $\alpha\beta > 1$, the results are reversed. In this case, if the economy starts from any point above the 45–degree line where $\mu \prec \pi^e$, the expected rate of inflation is rising. If the economy starts from any point below the 45–degree line where $\mu \succ \pi^e$, the expected rate of inflation is falling. In this case, point A is unstable and point C is locally stable.

Similar analysis may be carried out with respect to curves G′ and G″. The unique steady state at point E on G′ is stable from below if $\alpha\beta < 1$ and stable from

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[6] This assumption is reasonable, since inflation affects welfare inversely. For further details concerning the welfare cost of inflation see, for example, Cooley and Hansen (1991), Eckstein and Leiderman (1992), Gillman (1993), and Braun (1994).

above if $\alpha\beta > 1$. The curve G″ leads either to accelerating inflation or deflation depending on whether $\alpha\beta$ is less than or greater than 1.

As mentioned earlier, any exogenous change in seigniorage causes the G–curve to move to the left or right. If the government increases the budget deficit permanently, for example from S to S^*, in Figure 2, then the curve G shifts to G′. Consider the case, for example, where $\alpha\beta < 1$ and assume that the economy starts at the stable steady state point A. The change in seigniorage causes the economy to move to point F, with a jump in the growth rate of money. Since $\mu \succ \pi^e$ and $\alpha\beta < 1$, there is a gradual further upward movement of the expected rate of inflation and the growth rate of money as the economy moves from F to the new unique steady state at E.

The effects of an exogenous change in seigniorage can also be considered when $\alpha\beta > 1$. For example, consider that the economy starts in the stable steady state at point C for this case. Notice that point C is ‘on the wrong side of the Laffer curve’, that is point A, the unstable steady state, is preferable to point C since, point A shows lower inflation. A rise in seigniorage to S^* now causes the economy to move from point C to point D′, where $\mu \succ \pi^e$ and so $\dot{\pi}^e \prec 0$ from expression (13). The reduction of the expected rate of inflation causes the economy to move from point D′ to point E.

Thus, if $\alpha\beta < 1$ then, as long as seigniorage remains less than or equal to S^*, a rise in seigniorage will cause the stable steady state of inflation to rise. The reverse result is found when $\alpha\beta > 1$. If seigniorage exceeds S^* there is no steady state in either case.

Rational Expectations

The case of rational expectations is now examined, or, rather more accurately, of perfect foresight since the analysis has no random shocks. Following the perfect foresight assumption,[7] where $\pi_t^e = \pi_t$, any change in the anticipated sequence of deficits is immediately incorporated into inflationary expectations. Using this assumption, equation (12) becomes:

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[7] Rational expectations or perfect foresight can be considered as the limiting case when $\beta \to \infty$. Dividing both sides of equation (3) by *ß* and letting $\beta \to \infty$ yields $\pi = \pi^e$.

$$\pi = \frac{1}{\alpha}\ln(\mu c) - \frac{1}{\alpha}\ln S \tag{14}$$

and equation (7) yields:

$$\dot{\pi} = -\frac{1}{\alpha}(\mu - \pi) \tag{15}$$

Since *a* > 0, the analysis under perfect foresight is similar to the case when *aß* > 1 under adaptive expectations. However, in the case of rational expectations, inflation expectations can adjust instantaneously and the economy is assumed to jump immediately to the stable equilibrium because of the availability of information [(see, for example, Bruno et al., (1990)].

3 Seigniorage and Inflation: A Cash-in-Advance Model

3.1 The Microfoundation Model

A demand function for money is constructed by using the cash–in–advance model. Based on the Islamic Sariah most of the Islamic countries have prohibited the payment and receipt of a fixed or predetermined rate of interest. In the Islamic banking system the rate of return to financial assets held with banks is not known and not fixed prior the undertaking of each transaction. Profit- and loss-sharing arrangements are adopted in the system. Therefore, the percept influences the structure and the activities of their economics in general and their banking systems in particular and this paradigm affects macroeconomic policy and the performance of the economy.[8]

The model is based on the assumptions according to the characteristics of the economy as follows. The representative household holds two types of assets. They comprise money and trees. More specifically, trees indicate the investment deposits of the consumer in the bank as well as other kinds of assets that can be held by the household except bonds, for instance, durable goods and gold. The household holds the assets to share the profits of investment or to avoid money balances losing value as a result of inflation. There is no bond financing; consequently the issue of bonds is

[8] Forty-eight countries are increasingly involved in the Islamic banking. For example, Islamic Republic of Iran, Pakistan and Sudan are fully adopted Islamic banking principles. Other countries such as Malaysia, Indonesia, Bangladesh, Jordan, and Egypt operate Islamic banking alongside conventional banking. Furthermore, Islamic banking is expanding outside the Muslim countries such as the UK. For details see, for example, Ul–Haque and Mirakhor (1998), Errico and Farahbaksh (1998), and Sundararajan, Marston and Shabsigh (1998).

zero. There are two consumption goods, cash and credit goods. Government imposes a tax rate on labour income. A constant returns to scale technology is considered to transform labour into output.

The utility of the representative agent is a function over stochastic processes of two consumption goods, cash and credit goods, and labour. Consequently, the preferences may be written in the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, \ell_t) \tag{16}$$

where U (·) satisfies the usual concavity properties; ℓ_t is labour; c_{1t} and c_{2t} are the consumption of cash and credit goods, respectively. We use timing conventions as in Lucas and Stokey (1983). Cash goods can be purchased only with fiat money previously accumulated, while credit goods can be paid for with labour income contemporaneously accrued. The distinction between cash goods and credit goods may be motivated as follows. At some stores the buyer is known to the seller who is willing to sell on trade-credit, the bill to be paid at the beginning of the next period. The total amount purchased on this base is called credit goods. At other stores the buyer is unknown to the seller and there are resource costs involved in making oneself and one's credit-worthiness 'known' to someone else, so that the transaction is in terms of money. It is assumed that a current period utility function $U(c_{1t}, c_{2t}, \ell_t)$ follows a diminishing marginal rate of substitution between cash goods and credit goods.

The budget constraint can be written as

$$\begin{aligned} M_t + s_t(a_t p_t) = M_{t-1} - p_{t-1}c_{1t-1} - p_{t-1}c_{2t-1} + s_{t-1}(a_{t-1} + d_{t-1})p_{t-1} \\ + p_{t-1}(1 - \tau_{t-1})w_{t-1}\ell_{t-1} \end{aligned} \tag{17}$$

where s_t represents the trees[9] that the representative household holds; a_t is the price of trees and d_t is the return on trees that is called fruit; and w_t is the wage rate.[10] Moreover, the cash goods are restricted by the following standard cash–in–advance constraint:[11]

[9] See, for example, Lucas (1978).

[10] Braun (1991) considers w_t as a shock to labour's productivity, or a technology shock.

[11] Cooly and Hansen (1992) consider the following cash–in–advance constraint:

$$(1 + \tau_t)p_t c_{1t} \le m_t + (1 + R_t)b_t - b_{t+1}$$

where p_t is the price level; τ_t is the consumption tax rate; c_{1t} is the cash goods; $(1 + R_t)b_t - b_{t+1}$ is the principle plus interest from government bond holding, b_t; household acquires bonds that they carry

$$p_t c_{1t} - M_t \leq 0 \tag{18}$$

which indicates that household must have enough money to cover spending on cash goods. The aggregate resource constraint is

$$c_{1t} + c_{2t} + \leq w_t \ell_t \tag{19}$$

The consumer maximises expected utility in a stochastic environment, (16), subject to (17) and (18). Using the Lagrangean method, the maximand may be written as follows:

$$\begin{aligned} L = {} & u(c_{10}, c_{20}, \ell_0) + E_0 \beta u(c_{11}, c_{21}, \ell_1) + \cdots + \mu_{10}[p_0 c_{10} - M_0] + \mu_{20}[M_0 + s_0(a_0 p_0) \\ & - M_{-1} + p_{-1} c_{10-1} + p_{-1} c_{20-1} - s_{-1}(a_{-1} + d_{-1}) p_{-1} - p_{-1}(1 - \tau_{-1}) w_{-1} \ell_{-1}] \\ & + \mu_{11}[p_1 c_{11} - M_1] + \mu_{21}[M_1 + s_1(a_1 p_1) - M_0 + p_0 c_{10} + p_0 c_{20} - s_0(a_0 + d_0) p_0 \\ & - p_0(1 - \tau_0) w_0 \ell_0] + \mu_{12}[p_2 c_{12} - M_2] + \mu_{22}[M_2 + s_2(a_2 p_2) - M_1 + p_1 c_{11} \\ & + p_1 c_{21} - s_1(a_1 + d_1) p_1 - p_1(1 - \tau_1) w_1 \ell_1] + \cdots \end{aligned} \tag{20}$$

The first order conditions is derived by maximising equation (20) with respect to the consumer choice variables c_{10}, c_{11}, ..., c_{20}, c_{21}, ..., ℓ_0, ℓ_1, ..., M_0, M_1, ..., s_0, s_1, ….[12] The following relationships can be derived from the household's first order conditions:

$$\frac{u'(c_{1t})}{u'(c_{2t-1})} = \frac{p_t}{p_{t-1}} \tag{21}$$

$$-\frac{u'(\ell_t)}{u'(c_{2t})} = (1 - \tau_t) w_t \tag{22}$$

$$E_t\left[\frac{u'(c_{2t+1})(a_{t+1} + d_{t+1})}{a_{t+1}} \left(\frac{p_t}{p_{t+1}}\right)\right] = \frac{1}{\beta} \tag{23}$$

The representative preferences are specified as follows:

$$U(c_{1t}, c_{2t}, \ell_t) = \frac{\theta_1 c_{1t}^{1-\gamma_1}}{1 - \gamma_1} + \frac{\theta_2 c_{2t}^{1-\gamma_2}}{1 - \gamma_2} + \upsilon(\ell_t) \tag{24}$$

To derive the money demand function, the first order condition in equation (21) is used together with the derivatives of the representative preferences equation, (24), with respect to cash and credit goods. This yields:

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into the next period, b_{t+1}. It provides the household with $m_t + (1 + R_t) b_t - b_{t+1}$ units of currency for purchasing goods.

[12] For details see Moradi (1999).

$$\gamma_0 + \gamma_1 \ln(c_{1t}) - \gamma_2 \ln(c_{2t-1}) + \ln(\frac{p_t}{p_{t-1}}) = 0 \quad (25)$$

Using the cash–in–advance and recourse constraints the following money demand function is specified:

$$\ln(\frac{M_t}{p_t}) = \psi_0 - \psi_1 \ln(\frac{p_t}{p_{t-1}}) + \psi_2 \ln(Y_{t-1}) + \varepsilon_t \quad (26)$$

This model differs from the model derived by Braun (1994) in some fundamental aspects, in that the rate of inflation $\pi_t = \ln P_t - \ln P_{t-1}$ rather than the interest rate R_t which represents the opportunity cost of money. Moreover, in his model, bonds are one of the assets held by the representative agent, while in the proposed model tree is defined as an alternative asset.

3.2 The Empirical Model

The following model is considered for empirical estimation: [13]

$$m_t = c - \alpha \Delta P_t^e + \gamma y_t + u_t \quad (27)$$

where m_t is the log of the per capita real monetary base; ΔP_t^e is the expected rate of inflation, used as the opportunity cost of holding money; α is the semi–elasticity of the per capita real monetary base with respect to inflation, y_t is the log of per capita real GDP, and γ is the elasticity of the per capita real monetary base with respect to per capita real income.

The empirical money demand specification differs from the theoretical model derived in equation (26). First consider per capita real income. When the variables included in the model are cointegrated, y_{t-1} can be replaced by y_t, since in the long–run the difference between y_{t-1} and y_t can be ignored. Concerning the question whether actual or expected inflation should be included, the answer is that it does not matter for the cointegration analysis. Consider the relationship:

$$\Delta P_t = \Delta P_t^e + \omega_t \quad (28)$$

where ω_t is normally distributed with a mean of zero and a constant variance of σ^2. Provided the expectation error ω_t is stationary, it does not matter whether

[13] It should be noted that since 1979 the issue of bonds has been illegal in this economy. Although before 1979 there was bond financing, the proportion of income from bond financing was negligible.

expectations are rational or adaptive. Substituting ΔP_t from the above equation into equation (27) yields:

$$m_t - \gamma y_t = c - \alpha\Delta P_t + \upsilon_t \tag{29}$$

where $\upsilon_t = u_t + \alpha\omega_t$. Since ω_t is I (0), υ_t is also I (0) provided the original error u_t is I (0). Hence, if the original equation (27) is cointegrated then equation (29) will also be cointegrated.

Finally, a vector of dummy variables is included to consider the effects of domestic and international shocks as well as the government interventions.

Write the per capita demand function for real money as follows:

$$m^d = ce^{-\alpha\pi^e} y^\gamma \tag{30}$$

The aggregate demand for nominal monetary base is now:

$$M^d = NPce^{-\alpha\pi^e} y^\gamma \tag{31}$$

where N is population. Using the equilibrium condition, where money demand is equal to money supply and the steady state conditions $\pi^e = \pi$ and $\frac{d\pi}{dt} = 0$, and taking the logarithm of equation (31) and differentiating with respect to time yields:

$$\mu = n + \pi + \gamma g \tag{32}$$

where n is the growth rate of population and g is the growth rate of real income per capita.

Substituting the growth rate of money in equation (1) yields:

$$S = \frac{M}{P}\mu = \frac{M}{P}(n + \pi + \gamma g) = N.ce^{-\alpha\pi} y^\gamma (n + \pi + \gamma g) \tag{33}$$

To find the value of inflation that maximises the revenue from money creation, equation (33) is differentiated with respect to the inflation rate. The first order condition may be written as follows:

$$\frac{dS}{d\pi} = Nce^{-\alpha\pi} y^\gamma - Nc\alpha e^{-\alpha\pi} y^\gamma (n + \pi + \gamma g) = \frac{M}{P}[1 - \alpha(n + \pi + \gamma g)] = 0 \tag{34}$$

The rate of inflation that gives the maximum revenue satisfies the following equation:

$$\alpha(n + \pi + \gamma g) = 1 \tag{35}$$

The solution of this equation for the rate of inflation is

$$\pi^* = \frac{1}{\alpha} - n - \gamma g \tag{36}$$

As can be seen, the value of inflation that maximises the revenue, π^*, for developed countries is higher than developing countries, since in developing countries n and g are higher than developed countries. Using (32), the growth rate of money which maximises seigniorage revenue is:

$$\mu^* = \frac{1}{\alpha} \tag{37}$$

Substituting equations (36) and (37) into equation (33) yields the maximum steady state seigniorage revenue as:

$$S^* = \frac{Nc}{\alpha} e^{[\alpha(n+\gamma g)-1]} y^{\gamma} \tag{38}$$

Comparing equation (37) with (11), it can be seen that the inclusion of income and population does not affect the rate of monetary growth that maximises seigniorage. In contrast, the rate of inflation that yields the maximum revenue is influenced when income and population are also included in the model; the faster the rate of population or income growth the lower is the seigniorage maximising rate of inflation. The logic for this result is that government seigniorage revenue may be seen as being derived from two sources. One is the tax on existing real cash balances and the other is the tax on the additional balances that are demanded as population and income grow. Since the demand for cash balances declines as the rate of inflation increases, the revenue from the second source decreases as inflation rises. Thus, population or income growth causes the revenue maximising inflation rate to fall [(see, for example, Friedman, (1971)].

4 The Empirical Evidence

The data are annual over the 1961 – 1996 period. Population and official exchange rate (which is period average rate) were collected from International Financial Statistics of IMF, various issues. The other series were collected from the statistical books of the Central Bank of Iran.

Since there were extensive government subsidies, over the period 1961 – 1996, on consumer goods such as food, fuel and electricity, consumer price index (CPI) is unlikely to reflect the true inflation rate and, therefore, the GDP deflator is used in the study to calculate inflation. The parallel market exchange rate and the official exchange rate are defined as units of domestic currency required to purchase a unit of US dollar.

Macroeconomic Stylised Facts

Seigniorage is an attractive source for government finance in Iran. There is an inefficient tax system and collection costs are high, there is no advanced financial market, and the proportion of oil revenue in the government revenue is very high and volatile. Moreover, it should be pointed out that the oil price and the amount of oil exports of Iran are exogenously determined in the world market and OPEC, respectively. Consequently, the government uses seigniorage to smooth its spending, which demonstrates the link between fiscal and monetary policy.

This section considers some stylised facts of the Iranian economy concerning seigniorage and its relationship with macro variables. Figure 3 shows seigniorage obtaining from monetary base as a percentage of GDP over the period. Seigniorage has been on average 4.3 percent of GDP. There were spikes in seigniorage revenue following the first, second and third oil shocks in 1973, 1979 and 1986, respectively, the revolution in 1978 and exchange rate unification in 1993. The maximum seigniorage rate was 11.3 % of GDP in 1978 following the revolution.

Figure 4 provides useful insight concerning the analysis of seigniorage and inflation. As can be seen from the figure, the government has not always used this instrument efficiently. In some periods, the economy has remained on the wrong side of the Laffer curve where an increase in inflation is associated with a decrease in seigniorage revenue.

Figure 5 presents the relationship between the rate of inflation and the growth rate of nominal monetary base. As can be seen, these series move quite closely together over the period. Moreover, seigniorage shows a positive relationship with the growth of nominal monetary base (see Figure 6).

Figure 7 shows that a low level of expenditure is accompanied by a low level of revenue collection through seigniorage, and Figure 8 shows a positive relationship between the rate of inflation and real government expenditure as a percentage of GDP.

4.1 Order of Integration

Due to the presence of structural breaks in some time series, the Perron's procedure (1989) is applied to determine the degree of integration of the variables considered in this paper. Moreover, the augmented Dickey–Fuller (ADF) tests are applied to provide further information concerning the degree of integration of the time series. The order of lags in the regressions is chosen according to the Schwarz Bayesian Criterion, and the Akaike Information Criterion together with likelihood–ratio tests. Moreover, the lags are not omitted if their exclusion results in serial correlation.

Inspection of the plots indicates the presence of structural breaks in all series. Model (C) of the Perron procedure is used to test for unit roots in the level of the series and model (A) is used for the first difference. Table 1 report the results of the test for the levels and first differences. The results show that for all series there is a unit root in the level but not in the first difference. These results are consistent with the ADF results where the ADF results were not reported.[14]

Table 1 Univariate Perron Tests for Unit Roots in Levels and first Differences: 1961 – 1996

Levels		First Differences	
Series	I (d)	Series	I (d)
P_t	I (1)	ΔP_t	I (0)
m_t	I (1)	Δm_t	I (0)
y_t	I (1)	Δy_t	I (0)
E_t	I (1)	ΔE_t	I (0)
OE_t	I (1)	ΔOE_t	I (0)

Notes:

- P_t is the log of the implicit deflator of GDP at market prices; m_t is the log of the per capita real monetary base; y_t is the log of the per capita real GDP; E_t is the log of the parallel market exchange rate; OE_t is the log of the official market exchange rate; and Δ is the difference operator.

[14] Some other unit root tests such as Zivot and Andrews (1992) were used to endogenise the breakpoint. Here only the results of the Perron tests are reported.

4.2 Cointegration Tests

The Johansen–Juselius (1990) cointegration technique is used to estimate the per capita money demand function as specified in equation (27). The function includes two I (1) variables: per capita real balances and per capita real income; one I (0) variable: the rate of inflation; and a set of appropriate dummy variables. After considering all relevant dummy variables, only two were found significant: first dummy accounts for a one–time shift due to the revolution in 1978 and the second dummy accounts for an outlier in 1978.

The order of VAR (p) considered in the cointegration tests is determined on the basis of the information criteria provided by the AIC and SBC as well as LR tests.[15] The SBC and AIC suggest one and two lags, respectively. Since the likelihood ratio (LR) test suggested VAR (1) at 5 percent level, p = 1 is chosen.

The likelihood ratio statistics, λ_{max} statistic, is used in order to determine the number of cointegrated vectors. It confirms the presence of one cointegrating vector. It is concluded that there exists a long–run relationship between the variables when a break point in 1978 is accounted for.[16]

The results of ML estimates of long–run money demand function are as follows where the estimates are normalised to represent the per capita demand function for money:

$$m_t = -4.31\Delta P_t + 2.37 y_t \qquad (39)$$

where m_t is the log of the per capita monetary base; ΔP_t is the rate of inflation; and y_t is the log of the per capita real GDP. As can be seen, the signs of all coefficients are consistent with the theory. The semi–elasticity of inflation is 4.31, while the income elasticity is 2.37.

4.3 The Robustness of the Model and Estimates

To test the robustness of the per capita demand function for money, the log of the exchange rate (E_t) is included as an additional variable. The estimated equation of the per capita function shows that the coefficient of the parallel market exchange rate is

[15] Due to the small sample size, a maximum of two lags is employed.
[16] It should be noted that time trend is not significant in the models and so it is excluded.

insignificant in the model.[17] When the official exchange rate is used instead of the parallel market rate, the results confirmed that this is also insignificant. An ECM was also estimated, with Δm as the dependent variable, but the exchange rate was again insignificant.

The robustness of estimates is also investigated using other procedures. In the Johansen method, VAR (2) selected by the LR at 10 percent level is used to estimate the long–run coefficients. The semi–elasticity of the rate of inflation is 4.23, while the elasticity of real income 2.46. Comparing these results with those from VAR (1), the magnitudes of the values are very similar. Furthermore, when the VAR (1) is specified to include a trend, the results show that the trend is not significant in the model. The t–value of the trend is 1.19.

The autoregressive distributed lag (ARDL) method is also employed as an alternative method to specify and estimate the model. An ARDL (2, 2, 2) is considered. The long–run semi–elasticity of inflation is 4.64 and the income elasticity is 2.64, which are very similar to the estimates reported in previous sub-section. The ARDL method is also applied to sub–sample periods, pre–1978 and post–1978. This suggests the need to include the dummy variables D78 and TB78 to account for the break.[18]

The cointegrating regressions are also estimated using both OLS and the fully modified Phillips–Hansen (1990) methods.[19] The results again confirm that the signs of coefficients are consistent with theory, provided the model allow for the break in 1978.

Finally, the stability of the model is assessed using recursive estimation techniques. The results confirmed that the model is stable.

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[17] It should be mentioned that the parallel market exchange rate data for the period 1959 – 1993 collected from World Currency Yearbook, which were available, were also used, but the results did not change.

[18] Like the Johansen procedure based on the VAR, the ARDL method estimates the long–run effects jointly with the short–run effects. For small sample, as in the comparison of the sub–periods above, the ARDL method may be the most appropriate.

[19] Both of these methods estimate the long–run effects by ignoring the short–run adjustments. The P–H procedure makes an adjustment for simultaneity as well as autocorrelation and heteroscedasticity in the residuals. With OLS the standard errors are invalid even though the coefficient estimates are superconsistent in large samples.

4.4 Seigniorage–Maximising Rate of Inflation

This sub-section uses the empirical estimates of the demand function for the per capita real monetary base to calculate seigniorage–maximising inflation. The results for the whole and four sub–sample periods are reported in Table 2. It can be seen that the actual inflation rate exceeded the revenue–maximising inflation rate in the following periods:

- 1973 – 1978
- 1989 – 1996

The first of these periods followed the first oil boom, and the second period followed the end of the eight–year war. The latter period includes the implementation of the economic reform programme. The large value of π^* during the period of the war was due to the negative growth rate of per capita real GDP. The results imply that the government could have increased revenue through seigniorage by accepting a higher rate of inflation over the war period and by reducing inflation in the periods before and after the war.

Table 2 Actual and Seigniorage–Maximising Inflation Rates (Percent)

Period	g	n	π*	π	π* – π
1961 – 1996	2.18	2.85	15.19	13.09	2.10
1961 – 1972	7.30	2.88	3.00	0.42	2.59
1973 – 1978	1.77	2.86	16.15	19.75	-3.60
1979 – 1988	-4.70	3.63	30.37	15.49	15.24
1989 – 1996	3.40	1.82	13.31	24.09	-10.77

Notes:

- g is the growth rate of per capita real GDP; n is the growth rate of population; π^* is the seigniorage–maximising inflation rate, calculated from equation (36) using the estimates $\hat{\alpha}$ = 4.31 and $\hat{\gamma}$ = 2.37.
- π (= ΔP) is the actual inflation rate is calculated using $\Delta P = \ln P_t - \ln P_{t-1}$ measure.

The revenue–maximising inflation rate is also calculated period–by–period. Figure 9 plots the difference between seigniorage–maximising inflation and actual inflation rates. These results are consistent with those derived from the four sub–periods.

If we allow for the fact that the growth rate of the economy was negative during the war years, then the results suggest that $\pi > \pi^*$ following the first oil boom. This implies that the economy was on the wrong side of the Laffer curve, that is, point C in Figure 2. In order for this point to be locally stable, we need to assume that expectations are formed rationally or, if expectations are formed adaptively, that $\alpha\beta > 1$. To check for the last restriction, an ARIMA (0,1,1) model for inflation is estimated to obtain the value of β. The implied estimate of $\hat{\beta}$ is consistent with $\alpha\beta > 1$. Hence, the economy has operated at point C which is locally stable, irrespective of whether expectations are formed rationally or adaptively.

4.5 The Dynamics of the Model and Seigniorage Revenue

The error correction model (ECM) is also relevant here. Agents respond to changes in their equilibrium holding of real balances only gradually. Seigniorage revenue obtaining from monetary base can be decomposed into two parts. The first part of the revenue is generated from real balances in the new equilibrium and the second part is the flow of the revenue generated through the period, while agents are out of equilibrium [see, for example, Adam and et al., (1996)].

The error correction model is specified and estimated. After simplification the following model is obtained:

$$\Delta m = -1.20 + 0.04D78 + 0.21TB78 - 0.52\Delta P - 0.12EC_{t-1}$$

(-3.30) (0.99) (3.27) (-4.11) (-3.69)

n = 35 $\quad \overline{R}^2 = 0.685 \quad$ s = 0.060 $\quad \chi^2_{SC}(1) = 0.86$ [0.35]

$\chi^2_{FF}(1) = 0.92$ [0.34] $\quad \chi^2_{N}(2) = 1.95$ [0.38] $\quad \chi^2_{ARCH}(1) = 0.10$ [0.76]

where EC_t is the error correction term derived from the money demand function; n is the number of observations; $\overline{R}^2$ is the adjusted squared multiple correlation coefficient; s is the standard error of regression; χ^2_{SC} is Lagrange Multiplier (LM) test statistic for residual autocorrelation; χ^2_{FF} is RESET statistic for misspecification; χ^2_{N} is Jarque–Bera test statistic for normality; χ^2_{ARCH} is test statistic for autoregressive conditional heteroscedasticity; $D78_t$ takes the value of 1, if t $\geq$ 1979 and 0 otherwise and $TB78_t$ takes the value of 1, if t = 1978 and 0 otherwise. The numbers in brackets below the coefficients are t–statistics; and the numbers in square brackets next to diagnostic test statistics are P–values.

The crucial parameter of the ECM is the speed of adjustment coefficient. The low value of this coefficient, -0.12, implies that any deviation from equilibrium persists for a relatively long period of time. This has important implications for seigniorage revenue. Following a shock (e. g. an increase in inflation), agents adjust their real balances slowly toward new equilibrium and so agents hold excess money for some time. Hence, in this case, the government can generate considerable seigniorage revenue through the adjustment period.

The finding of a low adjustment coefficient may be due to the following interrelated facts:

- uncertainty,
- structural changes and exogenous factors,
- nonlinear structure.

The presence of uncertainty in the economy is important for the effectiveness of economic policies. Although measuring the impacts of uncertainty on the effectiveness of policy is difficult and it is beyond the scope of this study. Uncertainty implies that the policymaker cannot guarantee that his target value is attainable, since the target is affected by other factors in addition to policy actions. Obviously, the results of policy under uncertainty are different from the policy which would be pursued in a world of certainty [see, for example, Brainard (1967)].

The Iranian economy was dominated by movements in exogenous factors such as the oil price and various other shocks and government interventions. External shocks and some of internal shocks may be regarded as outside the government's control and they affect the economy and increases uncertainty.

However, although the government was able to earn revenue through seigniorage by accepting a higher rate of inflation, higher inflation resulted in higher uncertainty in the economy and affected the effectiveness of government policies.

Another explanation of slow adjustment may be that the true adjustment process is nonlinear as in Michael, Nobay and Peel, (1999). Their specification implies that adjustment is faster for large deviations from equilibrium.

An explanation of slow adjustment may also be that the true adjustment process is nonlinear as in Michael, Nobay and Peel, (1999). Nonlinear model of the demand function for monetary base is investigated to compare with the linear specification.

Following Granger and Terasvirta (1993), the nonlinear structure is investigated by testing linearity against two parametric nonlinear models: the exponential smooth transition autoregressive (ESTAR) and the logistic smooth transition autoregressive

(LSTAR) models. Granger and Terasvirta (1993) also note that the LM–type tests against a STAR model very often have good power even when the true alternative is a switching regression model [see, for example, Luukkonen et al. (1988a) and Peteuccelli (1990) for details]. However, the nonlinear model of monetary base is also generated a low adjustment coefficient.

5 Conclusion

This paper constructed a money demand function based on the cash-in-advance model and examined the relationship between seigniorage and inflation in Iran. There exists evidence of a Laffer curve relationship between seigniorage revenue and the rate of inflation. The evidence shows that the actual rate of inflation generally exceeded the corresponding rate that would maximise seigniorage revenue. This means that the government could have obtained extra seigniorage with a lower rate of inflation. However, during the war period the government could have accepted a higher inflation rate due to the negative growth in GDP. These results do not depend on whether the expectations of agents form adaptively or rationally.

Given the slow speed of adjustment of real money balances, the government can also generate considerable seigniorage revenue over the adjustment period. Uncertainty and structural changes and exogenous factors explain some parts of the low adjustment coefficient. The results of the slow speed of adjustment do not change substantially in the nonlinear method of estimations. Although the government was able to earn revenue through seigniorage by accepting a higher rate of inflation, higher inflation resulted in higher uncertainty in the economy.

Figure 3 Seigniorage Obtaining from Monetary Base as a Percentage of GDP

Figure 4 Seigniorage (as a percentage of GDP) against Inflation

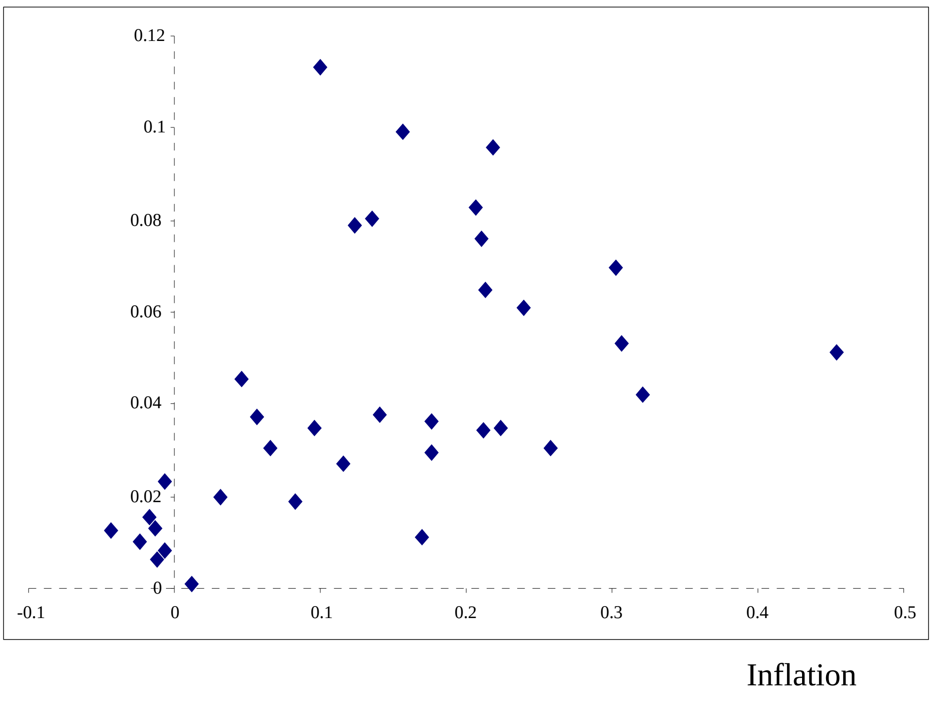


Figure 5 Inflation (DP) and the Growth Rate of Nominal Monetary Base (DMB)

Figure 6 Seigniorage (% GDP) and the Growth Rate of Nominal Money (DMB)

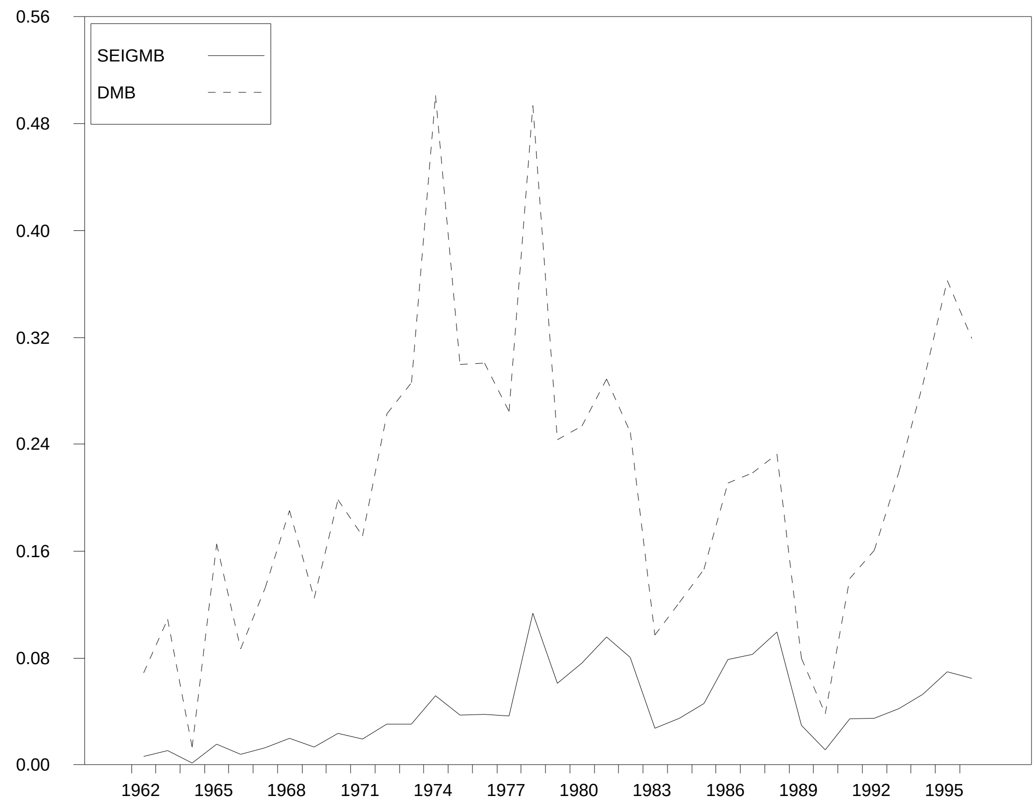


Figure 7 Seigniorage (SEIGMB) and Real Government Expenditure (RGE) as a Percentage of GDP

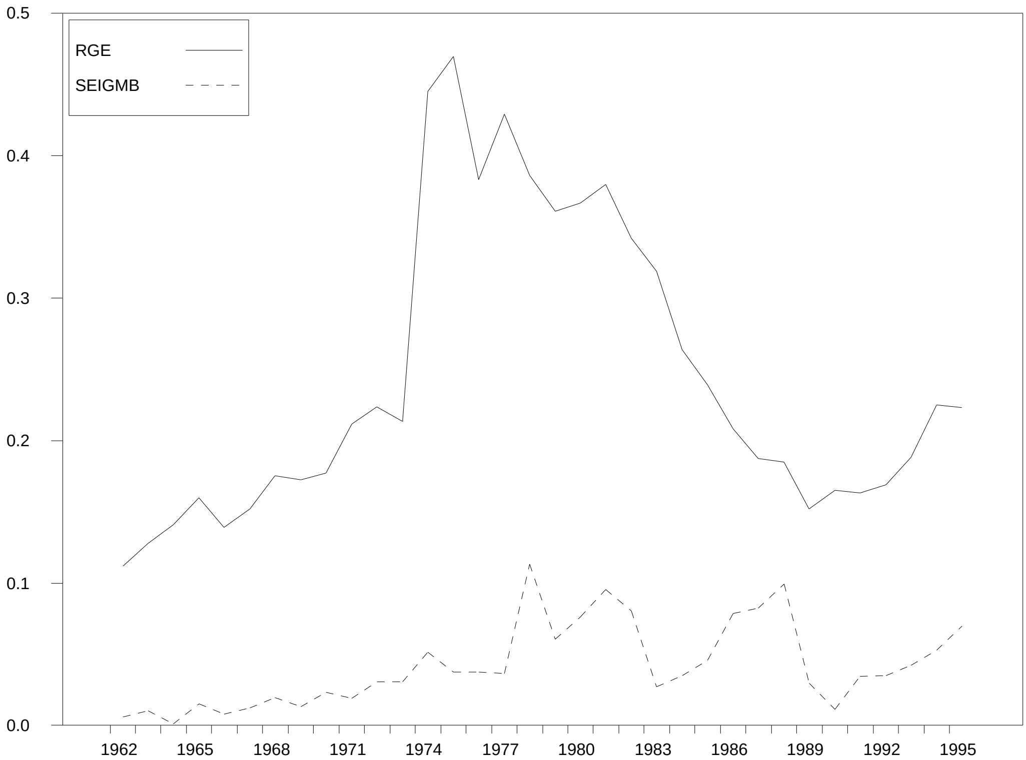
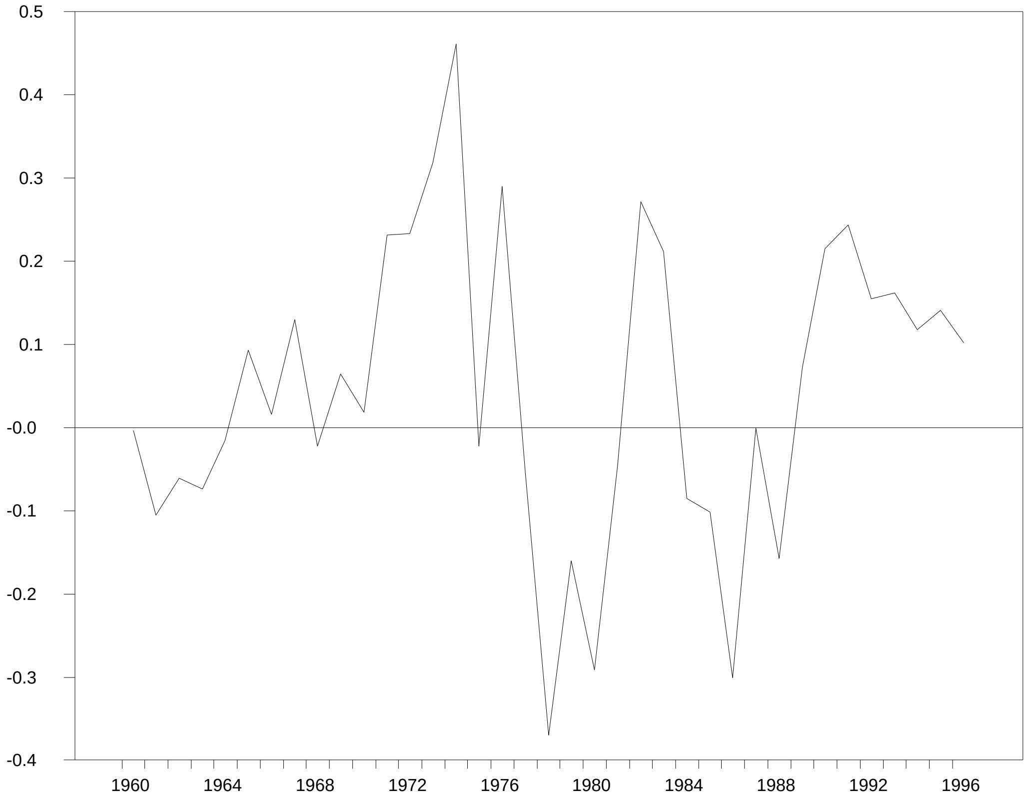


Figure 8 Inflation (DP) and Real Government Expenditure (RGE) as a Percentage of GDP

Figure 9 The Difference Between Seigniorage–Maximising Inflation and Actual Inflation Rates ($\pi - \pi^*$)



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